MATHEMATICAL METHODS OF SOLUTION PROBLEMS OF ELECTRODYNAMICS IN A LAYERED MEDIUM

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The article discusses methods for solving applied problems of electrodynamics in the case of layered and vertically inhomogeneous media. Algorithms for the primary processing of GPR data are proposed. The readings of a georadar at a fixed observation point were investigated using the "selection" method. The analysis of the frequency response of the signal energy distribution by frequency components is carried out. A software implementation for determining the depth of objects and the relative permittivity of the subsurface medium is proposed. To solve this problem, a module for determining the depth and conductivity of subsurface objects is used. The computational part was carried out by the method of an approximate solution that is widespread in computational practice. As a result of the study, numerical methods have been developed for solving direct problems of electrodynamics.

Keywords:*electromagnetic disturbance, class of functions, physical fields, numerical algorithms, physical characteristics, structure of media, direct problem, model equation, fixed point.*

Introduction. The actual and urgent need to use analytical and numerical methods for studying problems of electrodynamics determine the need to consider them in the case of layered and vertically inhomogeneous media. The original problem of electrodynamics, in the case of a special choice of the source of electromagnetic disturbance, is reduced to a series of one-dimensional problems for the geoelectric equation. Based on the "fit" method, fixed measurement points for a class of calculated physical fields determine a class of functions that describe the response of the medium.

To solve such applied problems, it is necessary to have a dependence of the signal amplitude on the depth of its reflection, and the initial radarogram will express the dependence of the signal amplitude on the reflection time. Then it is necessary to get rid of various noise hiding the useful signal.

Purpose of the work: For this purpose, algorithms for reducing the noise level in the radarogram using various wavelets should be considered. Used: Haar wavelet and Daubechies 4-order wavelets. The main field of application of wavelet transforms is the analysis and processing of signals and functions that are nonstationary in time and inhomogeneous in space. The results of such an analysis should contain the frequency response of the signal, the distribution of the signal energy over the frequency components. Compared to the decomposition of signals into Fourier series, wavelets are capable of representing local features of signals with a higher accuracy [1].

Main results. As a result of studying the subsurface environment, we will receive a lot of signals received from the receiving antenna for each measurement by the GPR. Many such traces are visualized using the variable density method as an image. Automation of computational processes should be implemented on the basis of a software module, which will allow determining the depths of objects and the relative dielectric constant of the subsurface medium. The location of the subsurface object is determined by the top of the hyperbola, which is constructed according to the points of the maximum values of the amplitudes of each trace [2].

To implement the set tasks, an algorithm is being developed to interpret the GPR data to determine the dielectric constant and conductivity of the medium. For this purpose, two practical inverse problems have been solved. At the beginning, the parameters of the source generated by the GPR are determined. According to a known source, the relative permittivity and conductivity of the medium are found. The tasks are solved on well-known environments that were specially prepared at the test site. For the first problem, a homogeneous medium was created, for the second - a local inhomogeneity located in a horizontally layered homogeneous medium. For the second task, a software module for determining the depth and conductivity of subsurface objects is described [3].

Experimental studies were carried out on a test site with a geological section containing perfectly clean sand and an inhomogeneous inclusion "salt dome" of artificial origin. The numerical algorithm allows one to determine the secondary source excited by a non-uniform inclusion and, subsequently, to determine the dielectric constant of this inclusion. The response of the environment, received from the GPR, was cleaned of noise and interference using filtering algorithms and wavelets. The tabular presentation of the environment response was used as additional information to solve the inverse problem of determining the geophysical properties of a localized object.

The results obtained demonstrate both the adequacy of the mathematical model and the possibility of practical application of the method under consideration for the interpretation of radarograms.

The original problem of electrodynamics, in the case of a special choice of the source of electromagnetic disturbance, is reduced to a series of one-dimensional problems for the geoelectrics equation [4]. Vertically inhomogeneous media are considered by us as cases of inclined media, as well as combinations of inclined and layered media, i.e. continuous (smoothed media) are considered. For such a case, there are no discontinuities in the coefficients of the physical characteristics of the media, which, in the case of discontinuities, lead to certain difficulties in constructing an algorithm for solving a straight line and, moreover, algorithms for solving inverse problems, in the latter case, in constructing a gradient of the residual functional. On the basis of the "fitting" method, the fixed measurement points for the class of calculated physical fields are determined by the class of functions that describe the response of the medium [5]. From the condition of the minimum square deviation of the observed field (GPR data at a fixed point) and the calculated physical field, by virtue of the uniqueness theorem, we obtain the required structure of the medium corresponding to the GPR data.

Consider an algorithm for a numerical method for solving the problem of electrodynamics in a layered medium. The "selection" method is a common method in computing practice for the approximate solution of an equation of the form:

$Az = u$, $\forall u \in U$, $z \in F$, where U, F aremetricspaces.

When using this method, given a sufficiently wide class of possible media, the corresponding calculated physical fields are calculated and, as a solution to the problem, some possible structure of the medium is selected for which the calculated physical field differs little from the observed field. An operator Az is calculated for an element z of some predetermined subclass of possible solutions $M(M \subset F)$, i.e. the direct problem is being solved. An element z_0 from the set M, on which the residual $\rho_u(Az_0,u) = \inf_{z \in M} \rho_u(Az,u)$ reaches a minimum is taken as an approximate solution [6]. In our case, the “selection” method is implemented as follows: let $M^{(1)}(\varepsilon_n,\sigma_n,\hbar_n^{(s)}), M^{(2)}(\varepsilon_n^{(s)},\sigma_n,\hbar_n), M^{(3)}(\varepsilon_n,\sigma_n^{(s)},\hbar_n)$ are the classes of possible media structures (geoelectric section), where $\hbar^{(s)} = \hbar^{(s-1)} + \delta\hbar^{(s)}$, $\sigma^{(s)} = \sigma^{(s-1)} + \delta\sigma^{(s)}$, s = 0,1,2 ... is the variation parameter, $\hbar$ is the width of the model layers. As noted above, we solve a series of direct problems $Az_j = u_j$, $j = \overline{1,n}$.. From where it is easy to determine the class of responses of the media, at a fixed observation point, i.e. $z_j(x,t) = g^j(t)$. Let the readings of the device (GPR) at the observation point be known, i.e. $f^j(t)$. According to the "fitting" method, we calculate the discrepancy $\rho_u^j(Az_j^*,u) = \inf_{z_j \in M} . \rho_u(Az;u_j)$ of the Element that delivers the minimum discrepancy is the response of the environment, from where the environment from the class of media is determined M^j, thereby solving the problem of interpreting the GPR data.

A set of algorithm solution data was created to construct a class of possible calculated physical fields for a set of geological section models in the case of layered media. Here are the ways of forming a class of possible structures of media, as shown in Figures 1-3.

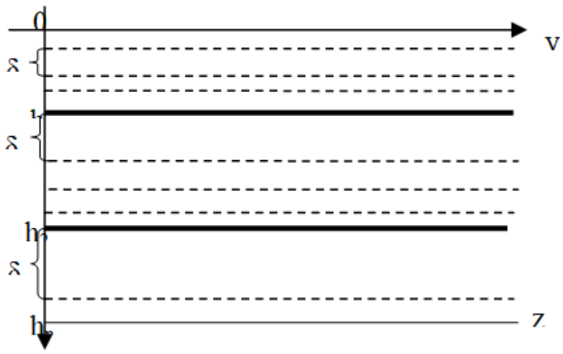


Figure 1 - Variations in depth of layer thickness

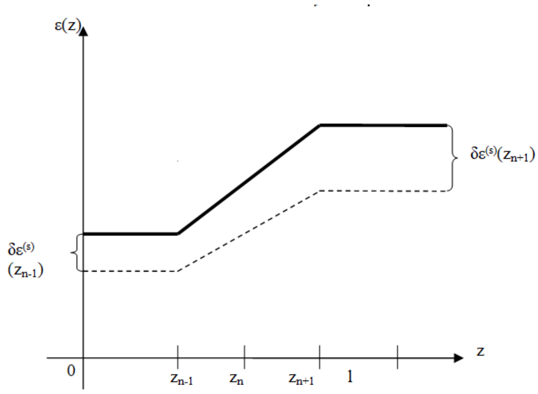


Figure 2 - Variations in the parameter-dielectric constant

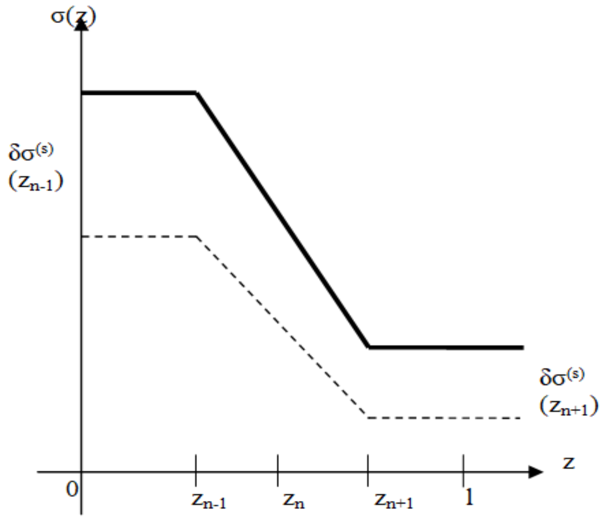


Figure 3 - Variations in the parameter-conductivity of the medium

By varying the main characteristics of the media (dielectric constant, conductivity of media, layer powers), a fairly wide class of possible media was created. Next, a series of direct problems were solved for each class of possible media structures.

For clarity of reasoning, we present the formulation of the direct problem of electrodynamics, which consists in the following: on the day surface, an external current source $\dot{j}^{cm}$ is switched on, which has a bell-shaped form in time $r(t)$. During about 30-50 nanoseconds, the response of the medium is measured, which is the solution of the direct problem at the point of observation (measurement).

We assume that the dielectric constant ε and conductivity σ depend on the depth x_3. Let us choose as a source of external current a sufficiently long cable located in the center and stretched along the axis x_2.

Under such assumptions, the system of Maxwell's equations is reduced to a system of one-dimensional problems in the constructed class and satisfies the following equations:

$$\varepsilon^{(s)}V^{(k)}{}_{tt} + \sigma V^{(k)}{}_{t} = \frac{1}{\mu}(V^{(k)}{}_{x_3x_3} - \lambda^2 V^{(k)}) - P_\lambda q(x_3) r'(t) \qquad (1)$$

$$V^{(k)}{}_{|t=0} = 0, \; V^{(k)}{}_{t|t=0} = 0. \qquad (2)$$

here: $\varepsilon = \varepsilon_0 \cdot \varepsilon_{отн}$, - dielectric constant, $\mu = \mu_0 \cdot \mu_{отн}$ - magnetic permeability, σ - medium conductivity $p(x_1)$, $q(x_3)$ - functions describing the transverse dimensions of the source, s - variation parameter, $V^{(k)}$ - solutions corresponding to the classes $M^{(k)}(\varepsilon_n^{(s)}, \sigma_n, \hbar_n), k = 1,2,3, m, e$.

Where: $V^{(k)} = F_{x_1}\left[E_2^{(k)}(x_1, x_3, t)\right]$, $P_\lambda = F_{x_1}\left[p(x_1)\right]$, λ - Where: - the parameter of the Fourier transform in the variable x_1.

$$\vartheta = p_\lambda q(x_3) r'(t), \quad \varepsilon = \varepsilon_0 \varepsilon_{отн}, \quad \mu = \mu_0\, \mu_{отн}, \; \varepsilon_0 = 8.854 \cdot 10^{-12} \frac{\Phi}{M}, \quad \mu_0 = 1.257 \cdot 10^{-6} \frac{\Gamma}{M}$$

Consider the case of a layered medium with known interfaces. In this case, we add to system (1) and (2), the continuity conditions for the horizontal component E_2, at the interfaces x_3^m

$\left[V^{(k)}\right]_{x_3=x_3^m}=0, \left[V_{x_3}^{(k)}\right]_{x_3=x_3^m}=0,$ m - break node number (3)

Formulation of the direct problem: Using the known values of piecewise constant functions $\varepsilon^{(s)}(x_3)$, $\sigma^{(s)}(x_3)$, and a positive constant μ, determine the function $V^{(k)}$ as a solution to the generalized Cauchy problem from relations (1) and (3). When using the class $M^{(3)}(\varepsilon_n, \sigma_n^{(s)}, \hbar_n)$, we consider a system of one-dimensional problems:

$$\varepsilon V_{tt}^{(3)}+\sigma^{(s)}V_t^3=\frac{1}{\mu}(V_{x_3x_3}^{(3)}-\lambda^2V^{(3)})-P_\lambda q(x_3)r'(t)$$

And accordingly, when using the class $M^{(1)}(\varepsilon_n, \sigma_n, \hbar_n^{(s)})$, we consider the following equations:

$$\varepsilon V_{tt}^{(1)}+\sigma V_t^{(1)}=\frac{1}{\mu}(V_{x_3x_3}^{(1)}-\lambda^2V^{(1)})-P_\lambda q(x_3)r'(t)$$

Here we carry out a variation in the thickness of the media layers and $\hbar^{(s)}=\hbar^{(s-1)}+\delta\hbar^{(s)}$.

Let us present a numerical algorithm for solving the direct problem, constructed according to the general theory of difference schemes [7].

We introduce a change of variables $\tau=\beta t, \beta$ - the dimensionlessness coefficient. Suppose that $\beta=10^8$, then the solution to the problem in new variables (τ, x_3), U : will take the form:

$$b^{(s)}U_{tt}^{(1)}+a^{(s)}U_\tau^{(1)}=U_{x_3x_3}^{(1)}-\lambda^2U^{(1)}-\gamma\overline{\vartheta}, \quad x_3\neq x_3^k \qquad (4)$$

$$U^{(1)}{}_{|\tau=0}=0, \qquad U^{(1)}{}_{\tau|\tau=0}=0 \qquad (5)$$

$$\left[U^{(1)}\right]_{x_3=x_3^k}=0, \qquad \left[U_{x_3}^{(1)}\right]_{x_3=x_3^k}=0 \qquad (6)$$

Where:

$b^{(s)}=c\cdot\varepsilon_{отн}^{(s)}$, $a^{(s)}=\gamma\cdot\sigma_{отн}^{(s)}$, $\gamma=1{,}256\cdot100$, $c=8.854\cdot1.256\cdot0.01$, $\overline{\theta}=p_\lambda\, q(x_3)r'(\tau)$.

Let us determine the size of the area of calculations by the variables x_3, and τ . For this purpose, we calculate the travel times of the direct and reflected waves in the media. We calculate the wave velocity by layers as follows:

$$v^{(k)}=\frac{1}{\sqrt{\varepsilon_0\,\mu_0\,\varepsilon_{отн}^k}}, \qquad \text{(k is the layer number)}$$

The difference scheme for equation (4) has the form:

$$b_i^{(s)}\frac{\hat{y}_i^{(1)}-2y_i^{(1)}+\overset{\vee}{y}_i^{(1)}}{\tau_0^2}+a_i^{(s)}\frac{\hat{y}_i^{(1)}-\overset{\vee}{y}^{(1)}}{2\tau_0}=\frac{1}{i}\left(\frac{y_{i+1}^{(1)}-y_i^{(1)}}{h_{i+1}}-\frac{y_i^{(1)}-y_{i-1}^{(1)}}{h_i}\right)-\lambda^2 y_i^{(1)}-\gamma\vartheta$$, if $i\neq i^k$, and

$$i=-N_1,-N_1+1,...,0,1,...,N_1,\quad j=3,4,...,N_2 \tag{7}$$

A difference analogue of the initial conditions (5):

$$y_i^{(1)}=0,\quad y_i^{(2)}=0,\quad i=-N_1,-N_1+1,...,0,1,...,N_1. \tag{8}$$

For calculations in a finite region, it follows from condition (6) that:

$$\hat{y}_{-N_1}^{(1)}=0,\quad \hat{y}_{N_1}^{(1)}=0,\qquad j=3,...,N_2, \tag{9}$$

Resolving equation (7) with respect to, we have:

$$\hat{y}_i^{(1)}=\left[\left(r_2y_{i+1}^{(1)}+r_1y_{i-1}^{(1)}\right)+\left(2b_i^{(s)}-\left(r_2+r_1+\lambda^2\right)\right)y_i^{(1)}-\left(b^{(s)}-0.5\tau_0a_i^{(s)}\right)\overset{\vee}{y}_i^{(1)}-\gamma\tau^2\overline{\vartheta}\right]\Big/\left(b^{(s)}+0.5\tau_0a_i^{(s)}\right)$$
$$i\neq i^k,\quad j=3,4,...,N_2. \tag{10}$$

Indicated: $r_2=\tau_0^2/i^hi+1,\quad r_1=\tau_0^2/i^hi$ at the rupture nodes, i.e. at $i\neq i^k$. Based on the conjugation conditions (6), we have:

$$\hat{y}_{i^k}^{(1)}=\left(\frac{1}{h_{i^k+1}}\hat{y}_{i^k+1}^{(1)}+\frac{1}{h_{i^k}}\hat{y}_{i^k-1}^{(1)}\right)\Big/\left(\frac{1}{h_{i^k+1}}+\frac{1}{h_{i^k}}\right),\qquad j=3,4,...,N_2. \tag{11}$$

Let's approximate the source $\overline{\vartheta}=q(x_3)r'(\tau)$. Let's put:

$$q(x_3)\cong\begin{cases}\cos\left(\pi\left(x_3/l_0+1\right)\right)+1, & x_3\in[-l_0,0]\\ 0, & x_3\notin[-l_0,0]\end{cases}$$
$$r'(\tau)\cong\begin{cases}(\pi/2t_0)\sin(\pi\tau/t_0) & \tau\in[0,t_0]\\ 0, & \tau\notin[0,t_0]\end{cases} \tag{12}$$

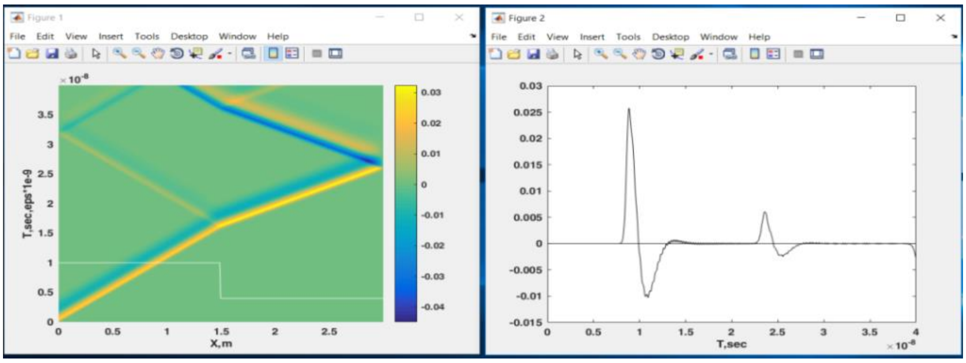
The values t_0 are determined from the condition of the problem, i.e. if the source duration is *2нс*, then for a real model $2t_0=2нс$ and in dimensionless form it will be 0.2 units. In our case, the Courant conditions have the form:

$$\tau_0<h_0/\overline{c},\ \text{where}\ h_0=\min_{-N_1<i<N_1}h_i,\ \overline{c}=\max_{-N_1<i<N_1}b_i$$

The area in time τ is approximated by a uniform mesh:

$\omega_\tau=\{\tau=(j-1)\tau_0,\ j=1,2,...,N_2\}$ where $N_2=\tilde{T}/\tau_0+1$, $\tilde{T}$ - travel time of direct and reflected waves. We approximate the region in terms of a variable x_3 with a non-uniform mesh so that the nodes of the discontinuities x_3^k coincide with its nodes $\tilde{\omega}_h = \{x_{3,i} = ih_i,\quad i = -N_1, -N_1+1, \dots, 0, 1, \dots, N_1\}$.

To demonstrate the operation of algorithms (7) and (12), Figure 4 shows the numerical solution of direct problems for a possible structure of a layered medium.



a) signal propagation b) medium response in a layered medium

Figure 4 - Numerical solution of direct problems for a possible layered media structures

Conclusion. The obtained GPR readings at a fixed observation point using the "fit" method were compared with the class of possible structures, created for more than 10,000 variants. Thus, the interpretation of the radarograms was carried out. Figures 6 and 7 clearly show the operation of this algorithm.Figure 6 shows the solution of the direct problem for the possible structure of a layered medium with a thick black line, a thin black line shows the readings of the GPR at a fixed observation point, obtained on 5002 iterations of the "selection" method. A similar result obtained for 6752 iterations is shown in Figure 7.

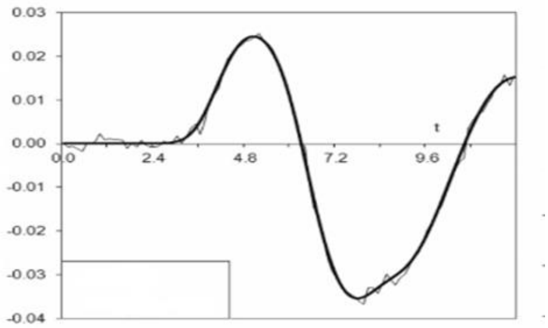


Figure 6 - The result of the "selection" method for the case of layered media

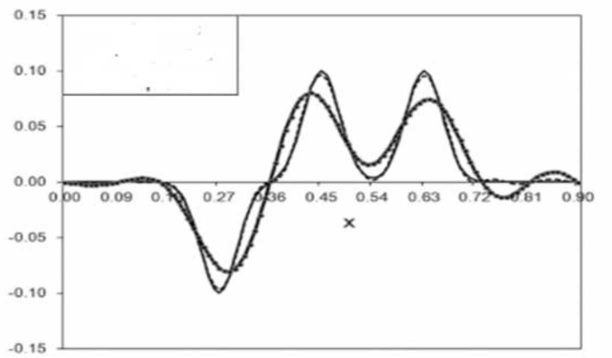


Figure 7 - Result of the "selection" method for the case of layered media

As a result of the study, numerical methods have been developed for solving direct problems of electrodynamics (layered media). Algorithms and a program of numerical methods for solving direct problems of electrodynamics have been developed. To compare the GPR data with the results of calculations of model problems for the geoelectric equation in the case of layered media, the “selection” method was used. In the class of finite-parametric media, an algorithm and software are built to determine the class of computed physical fields. Further, comparing the measurement data with this class, we reconstruct the geological section.

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